DRAG DUE TO LIFT: Concepts for Prediction and Reduction

Ilan Kroo
Department of Aeronautics and Astronautics, Stanford University, Stanford, California 94305; e-mail: kroo@stanford.edu

Key Words induced drag, vortex drag, winglets, nonplanar wings, aerodynamic design

Abstract This article describes some of the fundamental ideas underlying methods for induced-drag prediction and reduction. A review of current analysis and design methods, including their development and common approximations, is followed by a survey of several approaches to lift-dependent drag reduction. Recent concepts for wing planform optimization, highly nonplanar surfaces, and various tip devices may lead to incremental but important gains in aircraft performance. Focusing on relatively high-aspect-ratio subsonic wings, the review suggests that opportunities for new concepts remain, but the greatest challenge lies in their integration with other aspects of the system.

INTRODUCTION

It has been noted that transportation is fundamentally 0% efficient as it involves moving mass from rest at one point to rest at another point, so that the energy of the system is unchanged. That it does take energy to accomplish this objective is due to the presence of drag, and the reduction of drag has been a primary focus of aircraft design over the last century. At present, airlines pay hundreds of millions of dollars in fuel costs annually, and the environmental impact of aircraft is closely tied to the amount of fuel required, so the accurate estimation and reduction of this drag is of great interest.

At subsonic speeds, drag is caused by two basic phenomena: the influence of viscosity, primarily through skin friction, and losses associated with the generation of wing lift. Aircraft drag tends to vary roughly according to the equation $D = k_1 q + k_2 L^2 / q$, where $k_1$ and $k_2$ are related to the aircraft geometry; $L$, the lift, is equal to the aircraft weight in level flight; and $q$ is the dynamic pressure, $1/2 \rho V^2$. This simple relation implies that the minimum drag for a given lift occurs at a speed for which the two terms are equal. Thus, the drag associated with lift accounts for roughly half of the total drag when the aircraft is flown at conditions...
leading to maximum lift-to-drag ratio. Of course, this is rarely the case because higher speeds lead to improved productivity and, for turbofan engines, to improved engine efficiency, but nevertheless, drag due to lift constitutes approximately 40% of the total drag of typical transport aircraft at cruise conditions. At low speeds the situation is changed—drag due to lift constitutes most of the drag. In fact, a typical commercial aircraft might cruise at a lift coefficient of 0.5 and take off at a lift coefficient of 2.0. If the zero-lift and lift-dependent drag components were of similar magnitude during cruise, the drag due to lift would constitute more than 90% of the drag at takeoff. With higher viscous drag coefficients at the takeoff conditions, we still find that at takeoff the lift-dependent drag accounts for 80–90% of aircraft drag. Although one might argue that takeoff constitutes a very small portion of the flight, its influence on the overall aircraft design is profound. Because conditions associated with engine-out climb shortly after takeoff are often critical constraints in the aircraft design, changes in aircraft performance at these conditions influence the overall design and so have an indirect, but powerful, effect on the aircraft cruise performance. Although a 1% reduction in drag due to lift may improve the cruise lift-to-drag ratio by 0.4%, with a similar effect on range, the improved low speed climb performance may make it possible to achieve acceptable takeoff and climb with almost 1% greater takeoff weight, leading to an increase in range several times that associated with the simple cruise lift-to-drag ratio improvement. As a result, drag due to lift has a much greater significance to aircraft performance than might be inferred simply from the aircraft cruise aerodynamics. Furthermore, even for aircraft that are not constrained by a required climb gradient, lower drag at high lift conditions means reduced takeoff noise. Some types of aircraft also gain disproportionately from drag-due-to-lift reduction, including aircraft such as long-endurance autonomous air vehicles, for which lift-dependent drag is more important to performance.

This review deals with the prediction of wing drag due to lift and possible means by which it may be reduced. The discussion focuses on low-speed aircraft, although the ideas described here are applicable to certain aspects of higher-speed aircraft, automobiles, sailboat sails and keels, and many other devices.

Aircraft drag is often expressed simply in dimensionless form as follows:

\[ C_D = D/qS = C_{Dp} + C_L^2/\pi AR e + \Delta C_{Dc}, \]

where \( S \) is the wing reference area; \( AR \) is the aspect ratio, the ratio of span \( b \) to the average chord \( (AR = b^2/S) \); and \( e \) is the span efficiency factor (see Shevell 1983).

The first component of the drag, \( C_{Dp} \), is sometimes termed parasite drag and consists principally, but not entirely, of skin friction and pressure drag associated with viscosity. The second term varies roughly quadratically with the lift coefficient and is often called lift-dependent drag, and the third term, which is important for transonic aircraft, is related to compressibility. Although this decomposition of drag is widely used, its simplicity is a bit deceiving. Viscous effects, including skin friction, may affect the value of \( e \) in the lift-dependent drag term, whereas three-dimensional inviscid phenomena contribute to \( C_{Dp} \). The compressibility drag
is also lift dependent. This drag bookkeeping problem is exacerbated when propulsion systems are included. Because of the importance of lift-dependent drag at low speeds, and because the lift-dependent drag is dominated by the induced or vortex drag associated with the trailing vorticity shed by a three-dimensional lifting surface, this review concentrates on this component of wing drag.

The decomposition of drag into components tied to specific physical phenomena is especially important for design, for which results from simple analyses can provide an indication of what is possible and how closely a theoretical minimum is approached. This has certainly been true in the case of wing lift-dependent vortex drag, for which simple theoretical results have been available since the earliest days of aircraft development. Achieving minimal vortex drag has been a major concern of aircraft designers, and estimating the theoretical minimum, subject to a variety of constraints and simplifying approximations, has been an important topic in aerodynamics since its inception. That it remains an area of great interest reflects both the importance and the complexity of this topic.

Previous Reviews

The computation and reduction of lift-induced drag is of critical importance in aircraft design. Not surprisingly, then, many reviews of this field have appeared over the past few years. These generally fall into three basic categories, which parallel the sections of the present article: general reviews describing the history of the field and implications for aircraft, birds, or other vehicles; theory reviews, which describe analytic, computational, or experimental techniques for addressing drag due to lift; and summaries of specific drag reduction concepts. Some recent general reviews include a summary of recent work on drag prediction at Office National d’Etudes et de Recherches Aerospatiales (ONERA) (Schmitt & Destarac 1998), a history of induced drag research (Henderson & Holmes 1989), a review of the development of finite wing theory (Eppler 1987), and a summary of drag reduction concepts in nature, including the role of lift-dependent drag (Bushnell & Moore 1991). Several relevant papers are included in two Advisory Group for Aerospace Research and Development (AGARD) special courses on drag prediction and reduction, in 1977 and 1985, including a wide-ranging summary of approaches to drag reduction (Thomas 1985), a view of promising technologies for future transport aircraft (Poisson-Quinton 1985), and an overview of aircraft drag reduction concepts (Hefner & Bushnell 1977). Reviews of the relevant fluid mechanics and computational approaches include a recent summary of far-field drag methods for computational fluid dynamics (van Dam 1999), a related summary by Cummings (1996), a fundamental analytical look at the causes of lift-dependent drag (Yates & Donaldson 1986), and a closely related review of the fluid mechanics of airplane-trailing vortices (Spalart 1998). Some of the early history of induced drag computation is described in intriguing detail in Anderson’s (1997) recent book on the history of aerodynamics. Several papers in the proceedings of a workshop held at the National Aeronautics and Space Administration (NASA)
Langley facility in 1996 (see Bushnell 1996) deal with specific concepts for drag-
due-to-lift reduction, and Rokhsaz (1993) provides a survey of more than a dozen
wing tip devices.

FUNDAMENTALS

Before the development of the relevant theory, it was understood that a compo-
nent of the drag produced by a lifting wing varied roughly quadratically with the
lift produced at a given speed. This relationship was cited by Chanute (1894)
and Lilienthal (1889) based on the assumption that pressure forces act normal
to a flat plate. In this case, $C_D \approx C_{D0} + C_L \alpha$. The angle of attack, $\alpha$, required
to produce a given lift was determined empirically. This equation leads to much
higher lift-dependent drag than would actually be achieved by a well-designed
high-aspect-ratio wing, but measurements at that time with thin sections at very
low Reynolds numbers may have yielded results not too different from that given
by this expression. Unlike many researchers at that time, Lilienthal recognized the
importance of the aspect ratio, noting that "...it is evidently not immaterial whether
an obliquely placed, rectangular surface is pushed through the air lengthwise or
sideways. Though the two surfaces A and B shown in plan are of equal area, have
equal inclinations and equal velocity, there is a difference between both as regards
the air resistance, which points to the formation of eddies to a greater extent on A
than on B."

The study of inviscid lift-dependent drag, here termed induced or vortex drag,
was begun by Lanchester (1907) and Prandtl (1918), with Lanchester describing
the phenomenon and suggesting a far-field view in terms of the energy left by the
vortex wake. Prandtl developed a quantitative model known as lifting line theory,
which was refined by his students Betz, Trefftz, and Munk (see Anderson 1997).

While providing a means of estimating the distribution of lift on a simple wing
with a given chord and twist distribution, the lifting line theory provided a rather
good model for the computation of induced drag (the term coined by Munk) when
the distribution of lift was known. It could easily be shown (Munk 1921) that the
minimum induced drag for a planar wing of fixed total lift and span was
achieved with a distribution of wake-induced downwash that was constant in the
far wake. This required an elliptical distribution of lift and resulted in an induced
drag coefficient given by $C_{Di} = C_L^2 / \pi AR$. The total drag of a wing might then
be written in dimensionless form as follows:

$$C_D = C_{Dp} + C_L^2 / \pi AR e,$$

where $C_{Dp}$ includes the viscous drag, and the factor $e$, sometimes called the
Oswald efficiency factor (Shevell 1983), accounts for nonoptimal loading and
lift-dependent viscous drag. The dimensional version appears as follows:

$$D = qSC_{Dp} + L^2 / q\pi b^2 e.$$
This form of the expression reveals that induced drag depends on the wing lift, speed, and span and not on the aspect ratio, a fact that is underappreciated because the dimensionless form is so well-known. (We also note the importance of using the same arbitrary reference area for the definition of aspect ratio, \( C_L \), and \( C_D \).)

The form of the above equations suggests a means by which one might infer the drag-due-to-lift efficiency of a given design. By fitting \( C_D \) versus \( C_L^2 \) and considering the slope, one can determine the values of the constants \( C_{D_p} \) and \( e \). This has been done for years using wind tunnel or flight test data and often works reasonably well in identifying problems with a design or test technique. There have been many cases in which such a technique has led to surprises, however. Sailplanes, with wing designs that closely approach the “ideal” elliptical load distribution, are found to achieve values of \( e \) between 0.70 and 0.80 (Brown 1972). This large difference from the theoretical maximum value of 1.0 for planar wings reflects the fact that lift-dependent drag is not just the vortex drag computed by Prandtl. This fact is readily observed in the drag polars of two-dimensional airfoil sections, which often also exhibit a quadratic drag polar, although vortex drag is not involved. The additional lift-dependent drag is, in this case, due to the increased skin friction and viscosity-related pressure drag and on the section upper surface as \( C_L \) is increased. Because this effect is relatively independent of the aspect ratio, its effect is most pronounced for high-aspect-ratio wings.

A second difficulty in the interpretation of the Oswald efficiency factor arises because vortex drag can exist even at zero lift. In this case, each portion of a twisted wing produces lift (in the positive or negative sense) and interferes adversely with the neighboring portion, although the net lift is zero. This effect can be significant for wings or systems of wings with large variations in incidence. Incorrectly attributing some of the vortex drag to the viscosity-dominated parasite drag at zero lift has sometimes led researchers to announce breakthroughs in lift-dependent drag reduction as the fit of \( C_D \) versus \( C_L^2 \) leads to higher-than-anticipated values of \( e \). Direct application of linear aerodynamic theory to wings with twist leads to a somewhat more general expression for vortex-induced drag:

\[
C_{D_i} = C_L^2 / \pi A R \mu + \theta C_L v + \theta^2 w = C_L^2 / \pi A Re_{inviscid}.
\]

For a particular choice of the twist distribution shape, the amplitude \( \theta \) can be chosen to minimize vortex drag at a given \( C_L \). For planar wings and with reasonable choices for the shape of the twist distribution, the result is generally very close to Prandtl’s minimum at the selected \( C_L \). If the twist is fixed, the polar can still be represented by the simple two-term expression for \( C_{D_p} \), but it must be remembered that \( e \) changes with \( C_L \) (especially at low values of \( C_L \)).

The above discussion relies on the phenomenological result that the drag coefficient varies quadratically with \( C_L \). To better understand ideas for vortex drag reduction, a more physical description of vortex drag creation is helpful.
Vortex Drag Sources (Near-Field and Far-Field Views)

If a vortex wake were shed from the trailing edge, one could compute the change in local onset flow at every point on the airfoil. However, the streamwise vorticity actually develops along the chord, so that the full trailing vorticity is reached at the trailing edge. One can compute this effect as well, but this introduces a level of complexity that was impractical in 1918 aerodynamic computations, so Prandtl ignored the variation of streamwise vorticity on the section itself and computed the change in local flow angle only at a single point on the section. The later extensions of lifting line theory by Weissinger (1947) and others included this same averaging of the effect of the wake, but they also included the effect of the finite extent of the lifting line itself and reproduced the correct lift curve slopes in both the high- and low-aspect-ratio limits (Jones 1990).

The view of vortices inducing velocities that modified the local flow was appealing and intuitive. Munk used these ideas extensively in his analysis of vortex drag in the classic work “On the Minimum Induced Drag of Aerofoils” (Munk 1921). This view led to some difficulties, however, that could be overcome by considering the problem in the far field. Munk (1981) would later comment, “My principal paper on the induced drag was still under the spell of Prandtl’s vortex theory. Everything that Prandtl said was correct, but it was not the right approach.”

Considering only the incompressible, inviscid case in which the vorticity is confined to a thin sheet in the flow, basic momentum theory suggests that the forces can be written in terms of quantities on the outer boundaries of the control volume shown in Figure 1:

$$ F = \int \int \rho V \cdot \hat{n} \, dS + \int \int \frac{\rho}{2} V^2 \hat{n} \, dS. $$

If the control volume is extended so that the box is very large, the pressure and momentum integral leads to the following expression for drag:

$$ D_i = \frac{\rho}{2} \int \int (v^2 + w^2 - u^2) \, dS. $$

Here the integral for induced drag involves only the perturbation velocities, $u$, $v$, and $w$, over the aft face of the box, termed the Trefftz plane.

Although the drag is simply expressed in terms of perturbation velocities in the far field, there are some difficulties in applying this concept. First, the velocities must be computed or measured over a very large area, and second, we must evaluate the velocities a large distance behind the wing.

The first difficulty may be overcome by projecting the wake from some near-field plane, aft of the lifting system, to the far field in the streamwise direction (see Kroo & Smith 1990). Assuming incompressible potential flow in the cross-flow plane, we can then write

$$ D_i = \frac{\rho}{2} \int \int (v^2 + w^2) \, dS = \frac{\rho}{2} \int_{\text{wake}} \Delta \phi \frac{\partial \phi}{\partial n} \, dl = \frac{\rho}{2} \int_{\text{wake}} \Gamma V_n \, dl. $$
where $\Gamma$ is the circulation at the corresponding station on the wing and $V_n$ is the wake-induced velocity normal to the wake trace.

This far-field relation between $D_i$ and $\Gamma$ explains why the greatly simplified lifting line concept worked so well for estimating induced drag. By evaluating the downwash at the start of a streamwise trailing vortex wake (whether at the quarter chord or trailing edge or anywhere else), we obtain half of the downwash at the Trefftz plane (because the wake extends infinitely in both directions from the Trefftz plane but only in one direction from its start), so

$$D_i = \int_{\text{wing}} \frac{l}{U} \, dy = \int_{\text{wing}} \rho U \frac{\Gamma}{U} \, dy = \int_{\text{trefftz}} \rho \frac{\Gamma}{2} \, dy.$$  

Computation of drag using lifting line theory proved very accurate when the lift distribution was specified. Successors to lifting line theory (e.g. Weissinger approaches and vortex lattice methods) worked in just the same way but did a better job of predicting the lift distribution for a given wing geometry. When surface panel methods and subsequent nonlinear solvers first appeared, the temptation was to simply integrate the surface pressures to find lift and drag. To achieve the accuracy of the simpler vortex lattice methods, an extremely dense surface paneling was required (Towne et al 1983), and soon Trefftz-plane-based drag computations were included in codes such as Boeing’s linear panel method A502 (PanAir) and full potential code (TranAir). For Euler solvers, the far field drag computation is still appealing, although unlike the irrotational codes, Euler and Navier-Stokes solvers usually keep track of the vorticity indirectly, and it is sometimes difficult to prevent
artificial dissipation of the wake sufficiently far downstream that the Trefftz plane results can be applied. For this reason, somewhat more complex expressions in the not-so-far field can be applied. The approach is discussed in more detail by van Dam (1999) and Cummings et al (1996).

Experimental Wake Surveys

Just as far-field information may be used to improve the accuracy of numerical drag estimates, experimental wake surveys have some advantages over conventional balances in the wind tunnel. As the use of automated and very small five- and seven-hole cone probes has become more common, wake surveys have been used to infer additional information about the drag due to lift. Starting with the work of Maskell (1973) and Wu et al (1979), and more recently with that of Brune (1994), attempts have been made to use wake information to separate the contributions of viscous and vortex drag. This has obvious advantages for design diagnostics, and researchers have reported some success along these lines, although, as mentioned by Spalart (1998), there is some tradeoff between apparent induced drag and the inferred viscous drag that varies with the streamwise location of the survey. This is partly related to the difficulty in finding a suitable definition of induced drag in real, viscous flows, particularly those with separation. However, in the case of simple attached flows, such techniques can provide some insight into and rather accurate estimates of the vortex drag (Shoemaker 1994).

More Subtle Phenomena

An important simplifying assumption in Prandtl’s wing model is that the vortex wake extends from the wing in straight lines parallel to the freestream. Because of its self-induced velocity, the wake does not actually move in the freestream direction. The difference in direction may not be large, but the cumulative effect leads to very different wake geometry in the far field. We, therefore, may be surprised that the simple straight wake model provides anything approximating the correct solution for induced drag. Indeed, if we compute the wake shape by considering the roll-up process and integrate in the far field for the induced drag, we find that the result is very sensitive to the wake shape. As shown by Smith (1996), one must accurately model the roll-up process to obtain correct results by far field wake integration. Yet the Prandtl model works well with a very poor representation of the wake shape. The explanation for this lies in the fact that a far field computation of the force will be correct only if the wake itself is force free. The correctly shaped, rolled-up wake is indeed force free, so we expect that this will yield the correct force. The straight wake assumed by Prandtl is not force free, but it is drag free, because forces can only be produced on the vortex wake in a direction perpendicular to the vortices. Thus, the far-field computation of induced drag yields the correct result with either of these two wake models: the correct but hard to compute rolled-up shape, or the simple streamwise shape assumed by Prandtl. We note that for planar wings, the streamwise wake is basically lift free.
as well, so the computation of lift in the far field is also acceptable. For nonplanar wings, extending the wake in the freestream direction leads to a wake that is not lift free, and therefore the simple far-field lift model may not be acceptable in this case. One subtle aspect of this result is that the streamwise projection of the vortex wake provides a good approximation only when the projection is made from a location sufficiently far downstream of the lifting system. In practice this usually means that the initial deformation of the wake should be included up to the most aft location of the lifting surface.

Multiple Surfaces

The same analysis may be applied to systems of wings, oriented in various directions. Munk (1921) addressed many aspects of such systems in NACA document TN 921. Still “under the spell” of Prandtl’s vortex models, Munk evaluated the induced drag of wing systems by computing the local induced velocity on each surface. He proved that in the interaction of two surfaces, the effects of the transverse or bound vorticity on induced drag canceled, and he showed how the interference drag on each surface varied as the longitudinal separation between elements was changed. One of the surprising results was that if the circulation of each element was held constant, the total induced drag of the system was unaffected by changes in the longitudinal positions of the elements. This has become known as Munk’s stagger theorem and is of great importance in evaluating concepts for induced drag reduction. On reflection, Munk’s results can be inferred much more simply by noting that longitudinal motions of individual lifting elements have no effect on the far field wake and, therefore, cannot affect the system’s vortex drag.

The stagger theorem can be used to simplify the computation of the induced drag of lifting systems. That swept wings with a given lift distribution have the same drag as unswept wings becomes apparent. With a fixed loading on the wing and tail, the total drag is independent of the tail length, in spite of the complex interference between the bound vorticity and trailing wakes of each element. We can also quickly infer Munk’s mutual induced drag theorem, which states that for an unstaggered system of two wings the interference drag associated with downwash on one wing caused by the other wing is equal to the interference drag on the second wing caused by the induced flow from the first. (This is left as a problem for the interested reader.) Because the minimum induced drag is achieved by a certain circulation distribution, as long as the elements of the system can be made to achieve this loading, the minimum induced drag of a system does not depend on the longitudinal arrangement of the elements.

Some of these classical induced drag results do rely on the simple straight wake models and an assumed linear relationship between lift and circulation, and some of the drag reduction concepts described in the next section exploit the discrepancies between actual flows and these simple models. Although the basic induced drag results apply to incompressible flows, the results are much more broadly applicable. This is partially related to the fact that the transverse velocities
in the far field are generally low, even when the freestream speed is high (Smith 1997).

DRAG REDUCTION CONCEPTS

History

Even before the mechanism of induced drag was fully understood, concepts for reducing this important drag component were proposed. As noted by Whitcomb (1994), Lanchester applied for a patent on wing end plates in 1897. A steady stream of publications and patents dealing with various concepts for induced drag reduction have appeared since then. From theoretical studies of wings with end plates, done as early as 1924 (Reid 1924, Hempke 1927, Mangler 1938), to computational studies performed in the 1960s and 1970s as vortex lattice methods became more accessible, and to recent system studies, drag reduction devices, especially wing tip devices, have remained popular topics in the aerodynamics literature.

The concept of eliminating wing tip vortices or, indeed, all trailing vorticity has probably been discussed since Lanchester first drew his well-known picture of vortex trunks, and it continues to be mentioned in news groups and even in more formal circles. That the lift can be related to this far-field vorticity means, however, that any attempt to eliminate the induced drag entails eliminating lift as well. More promising approaches to induced drag reduction involve more subtle modification of the flow field and, to an increasing extent, multidisciplinary considerations. The results of Prandtl and his students for the minimum induced drag of a planar wing of fixed span and lift serve as a baseline to which drag reduction concepts are compared.

Span

Perhaps the most direct way to reduce the lift-dependent drag of a wing is suggested by Prandtl’s elliptical wing result itself: $D_i = \frac{L^2}{\rho \pi b^2}$. One can easily reduce the induced drag of a wing by 10% simply by increasing the span by about 5%. The reasons that this is not often a practical solution are varied but worth considering.

In some cases such an approach is explicitly banned. From standard class sailplanes, America’s Cup boats, and racing cars, for which very specific measurements related to span are an explicit part of the regulations, to carrier decks and airports that can accommodate aircraft with spans no larger than existing aircraft, it is often the case that increasing the wingspan is not an option. Even in these cases, it is sometimes possible to bend the rules by using folding wing tips (as proposed for the Boeing 777) or flexible surfaces that satisfy the static requirements but exceed the specifications under load. In a recent large aircraft study, the span was explicitly limited by airport requirements and by the desire to manufacture single-piece wing skins using existing manufacturing facilities. These constraints are sometimes misleading. In another design study it was argued that
adding winglets and reducing the span of a small transport aircraft would enable
more aircraft to be parked at existing airport gates. When considered in detail,
it was found that these small aircraft were sometimes parked next to large air-
craft, with wings overlapping. The addition of winglets would aggravate the gate
problems more than would span extensions.

More often the argument against reducing induced drag through an increase in
span is related to structural weight. Increasing wingspan, with a fixed wing area,
increases the weight of the wing due to higher bending moments and a thinner, less
efficient structure. So although induced drag varies with $1/b^2$, the wing bending
weight varies as $b^3$. At some point, the extra weight of the wing structure offsets
the reduction in vortex drag. Suppose the wing area is fixed and the speed is fixed.
If drag varies as $D = D_0 + kW^2/b^2$ and weight varies as $W = W_0 + k_2 W_0 b^3$, then
$D = D_0 + kW^2(1 + k_2 W_0 b^3)^{2/b^2}$. Solving for the span that minimizes drag leads to
$W = W_0 + W_0/2$; that is, the wing weight should be one-third of the weight of the
airplane. So, although there is a tradeoff between vortex drag and wing structural
weight, the wings of most airplanes are far from the region where these trades are
even, since typical transport aircraft wings constitute closer to 10% of the aircraft’s
gross weight.

Jones (1984) mentioned this and suggested that increases in aspect ratio might
be warranted. The tradeoff is a weak one, however, with a very flat optimum, and
for a variety of reasons (including structural dynamics, airport compatibility, and
fuel volume) the true optimum does occur in the range of existing aircraft. This
means that despite the readily available opportunities for induced drag reduction
(through increased span), it is not in the best interest of the system as a whole to
pursue them. This conclusion changes, however, when new technologies change the
relative sensitivities of drag or structural weight to span. The problem of induced
drag reduction is clearly not an aerodynamic problem; it is a multidisciplinary
design problem. Improved composite materials, the use of strut-braced wings,
or the application of active maneuver load control systems reduces the penalties
associated with span and permits induced drag savings through direct span increase.
Because we are now in a much better position to evaluate these multidisciplinary
tradeoffs, using high-fidelity analysis and optimization tools, progress in vortex
drag reduction may be more feasible.

Recognizing the issues with wing structural weight, Prandtl and others, includ-
ing Jones, considered solutions for minimum induced drag in which the span was
allowed to change but the lift distribution was constrained so as to limit certain
parameters related to structural weight. Solutions by Prandtl (1933) for minimum
drag with a fixed integrated bending moment (intended as a surrogate for struc-
tural weight) and by Jones (1950) with a fixed root-bending moment showed some
surprising results. In each of these cases, it was shown that by departing from the
elliptical distribution of lift and using a distribution that supported less outboard
load, one could increase the span without changing the structural constraint. The
reduced span efficiency factor, $e$, associated with nonelliptical loading could be
more than offset by the span increase. Prandtl’s fixed integrated bending moment
solution yielded an 11% reduction in induced drag, despite a span efficiency of only 0.75. The required span increase of more than 20% posed some difficulties, and the introduction of twist or high wing taper would cause concern for off-design operation, but such wings were considered in some NASA and Boeing studies in the 1970s. Jones’ solution with a fixed root-bending moment is quite useful. One can write the maximum span efficiency in terms of the location of the lift centroid as follows:

\[
\frac{1}{e_{\text{max}}} = \frac{9}{2\pi^2 \eta_c^2} - 12\pi \eta_c + 9.
\]

When the centroid is at 4/3π (42.4%) of the semispan, the wing may be elliptically loaded. This result is perhaps of more interest in the design of sailboats, for which heeling moment constraints are important, or to swept tailless aircraft than it is in the design of conventional aircraft wings. In the case of tailless aircraft, which achieve trim through a combination of sweep and twist, the location of the lift centroid is an important design parameter. The Horten flying wings (Gyorgyfalvy 1960) were initially designed with a lift centroid at about 33% of the semispan. This led to a very acceptable lateral control response but limited the wingspan efficiency, based on the expression shown above, to just 0.72. Jones (1980) later applied this result to the problem of optimal flapping, for which the root moment was again of great importance.

In practice, the selection of an optimal lift distribution is more complex than the Prandtl or Jones solutions might suggest. When area is held constant and increases in span change the wing chord, additional span represents a much greater penalty than might be suggested by the fixed integrated moment solution. With more realistic representations of wing structural weight, we find that the optimal span is only a few percent larger than would be suggested with elliptical loading (Figure 2; Kroo 1984). In reality the situation is much more involved. Minimum gauge constraints change the importance of structural loads at different stations, and indeed the structural sizing of the wing depends not on the cruise loads but on the loading at tens or hundreds of other flight conditions. The selection of cruise span loading affects wing maximum lift, tail loads, and compressibility drag as well, requiring the use of high–fidelity, multidisciplinary optimization and making the evaluation of drag reduction concepts very difficult.

Planform Design

For a given span, lift, and speed, induced drag is minimized with an elliptical distribution of loading. The implications of this result for wing design are interesting. For decades after this result was published, aircraft wings took on an approximately elliptical planform, because according to the lifting line theory, such a wing would provide the minimum induced drag. Of course by twisting the wing one could obtain an elliptical load distribution with any planform, and by increasing the tip chords and adding washout (reduced tip incidence) one could improve stalling characteristics while maintaining an efficient cruise wing. Wings with linear taper were also easier to manufacture, and so even sailplanes adopted
trapezoidal or double-tapered wing planform shapes. Throughout the history of wing design, however, engineers have experimented (numerically and physically) with various planform shapes, especially tip geometries that might improve the drag characteristics of wings. Such studies are simplified by the approximations involved in lifting line theory, including the flat wake geometry, inviscid flow, and neglect of chordwise flow variation. This is highlighted by recent results (Smith & Kroo 1993) that apply modern analytical methods to simple wings, showing, among other things, that an unswept elliptical planform wing does not produce an elliptical load distribution. Figure 3 shows the planform shape with straight quarter chord line that leads to elliptical loading. The tip chords are larger than those of an

**Figure 2** Variation in drag with span with fixed structural weight.

**Figure 3** Planform of unswept, untwisted wing with elliptical loading.
elliptical planform wing owing to the approximations made in lifting line theory that tend to overestimate the tip load. (This overestimation also leads to somewhat larger estimated induced drag penalties for rectangular wings than are predicted by more refined potential methods.)

The wing tip planform has been a subject of debate and study for decades. In 1923, NACA researchers studied the effect of swept wing tips (Zahm et al. 1923) on lift and drag, and a flurry of publications in 1985–1995 applied more modern approaches to the study of swept, sheared tips (van Dam 1985, Vijgen et al. 1987, Burkett 1989, Fremaux et al. 1990, DeHaan 1990, Smith & Kroo 1993). The Boeing 767-400 planform reflects this continued interest in wing tip geometry with its unconventional tip shape (Figure 4).

Although some studies have suggested fundamental advantages for highly swept or sheared tips, and although a small increment in span efficiency might be expected owing to the increased loading associated with sweep, it appears likely that the principal advantages of such a design will depend on the specific application. The effects on maximum lift and the complexity of the high lift system on lateral control response and on aerostatically induced twist all contribute to the selection of wing tip planform for a new or derivative design.

The impact of nonaerodynamic and off-design considerations in planform and tip design is great and has been addressed in studies such as those by Wakayama and colleagues (Wakayama 1994, Wakayama et al. 1996). The importance of various constraints on the planform design is clear from the results of numerical computations.

**Figure 4** Boeing 767-400 wing planform.
optimization studies in that work, which included airfoil thickness distributions and chord and twist distributions as well as sweep and operational design variables. Significant changes in the optimal shape are introduced by considerations of aeroelastics (on off-design loads and control power), fuel distribution, landing gear integration, and high lift performance.

Nonplanar Systems

One of the most often-cited approaches to induced drag reduction is the application of nonplanar lifting surfaces. From endplates to winglets, spanwise camber to boxplanes, it has long been known that significant vortex drag reductions may be achieved with nonplanar systems compared with planar wings of the same span and total lift. This section deals with some of the general ideas behind induced drag estimation and minimization for nonplanar lifting surfaces, and subsequent sections describe specific concepts for drag reduction.

In linear aerodynamic theory, the solutions for minimum vortex drag can be computed in several ways. Often it is convenient to solve directly for the circulation distribution that leads to the minimum drag, subject to bending moment, lift, or trim constraints. When the objective is quadratic and the constraints are linear (as they are in several interesting problems), the quadratic programming problem can be solved with a single linear system solution. Alternatively, we may solve for the optimal downwash distribution in the Trefftz plane and then infer the ideal loading based on a Schwartz-Christoffel transformation of the geometry into a simpler system that can then be integrated analytically. The latter approach, taken by Jones & Lasinski (1980), has certain advantages, providing an understanding of the sources of drag reduction. In either case, we generalize the basic expression for far-field drag, based on the previous discussion, as follows:

\[ D_i = \int \omega \rho \left( \frac{V_{n Trefftz}}{2} \right) \Gamma dy \]

where \( V_{n Trefftz} \) is the component of induced velocity perpendicular to the trace of the wake in the Trefftz plane, here termed normalwash.

One of the most useful approaches to the solution of this problem is the method of restricted variations. Munk, Jones, and others used this approach for solving many induced-drag-related problems. The idea is to consider a small variation in the optimal circulation distribution that satisfies the specified constraints. In the simple case of a planar wing, we might consider a small variation, \( \delta \Gamma_1 \), at some location on the wing and another variation, \( \delta \Gamma_2 \), elsewhere on the wing. To maintain constant lift, we require that \( \delta \Gamma_2 = -\delta \Gamma_1 \). The change in drag can be simply evaluated to the first order by applying Munk’s stagger theorem. We treat the perturbations in circulation as small wings and move them far downstream. Because this does not affect the overall drag of the system, we can easily write the first-order effect of these perturbations on the system-induced drag:

\[ \delta D = w_1 \delta \Gamma_1 + w_2 \delta \Gamma_2 \]

For the original distribution of circulation to represent a
minimum drag solution, \( \delta D \) must equal 0. This means that \( w_1 \delta \Gamma_1 = -w_2 \delta \Gamma_2 \). Adding the constraint on lift leads to \( w_1 = w_2 \); in other words, the downwash in the wake behind the ideally loaded planar wing is constant, as shown by Munk in 1921. With the addition of a root-bending moment constraint, we find that the downwash should be distributed linearly over the semispan, whereas Prandtl’s problem with fixed integrated bending moments leads to a parabolic variation of downwash. When the wing is not planar, the constraint that total lift is preserved leads to the following relation:

\[
\delta \Gamma_1 \cos \theta_1 = \delta \Gamma_2 \cos \theta_2,
\]

and the result, again attributed to Munk, is that the optimal normalwash should vary with the cosine of the dihedral of the wing at that spanwise location, \( \Theta \).

One can easily apply this approach to study a wide range of possible nonplanar wing concepts, characterized by the shape of their (assumed streamwise) wake in the Trefftz plane. Figure 5 shows some examples of just what is possible for nonplanar systems as a function of height-to-span ratio. For fixed height and span, the minimum vortex drag is achieved with a box plane arrangement, as noted by von Karman & Burgers (1935).

Figure 6 shows how the shape of the wake affects the minimum drag by illustrating the maximum span efficiencies for a range of concepts with fixed height and span. Note that the vertical extent of the system near the tips is the critical parameter and that although the box plane represents the absolute minimum solution, many other concepts provide very similar drag reductions and show that spanwise camber is most effective near the tip (Lowson 1990).

It should be noted that these results represent inviscid solutions. Some studies have also included the effect of lift-dependent section drag in the optimization.
CONCEPTS FOR INDUCED DRAG REDUCTION

1.03
1.05
1.32
1.33
1.36
1.38
1.41
1.45
1.46

Figure 6  Span efficiencies for various optimally loaded nonplanar systems ($h/b = 0.2$).

(e.g. Rokhsaz 1992, Kroo 1984). Although the section polar usually has a small effect on the optimal loading, the role of even constant section drag has quite different effects for planar and nonplanar wings. If the section drag coefficient is considered constant and the wing area is fixed, the optimal lift distribution for a planar wing remains elliptical and the span grows without bounds (or until Reynolds number and structural considerations must be included). For nonplanar wings, however, if the projected area and lift are fixed (so that sections can operate at the same $C_1$ as in the planar wing case), adding nonplanar area adds to the total drag. The optimal lift distribution is then a function of the viscous drag, and there is an optimal winglet height, for example.

Multiple Surfaces

Biplanes represented an early example of nonplanar wings, and the potential for induced drag reduction was not lost on early aerodynamicists. By assuming that each wing was elliptically loaded, Prandtl derived a simple expression that was used to determine the induced drag of biplanes with arbitrary vertical gap and span ratio. Prandtl’s biplane equation is written as follows:

$$D_i = L_1^2/\rho \pi b_1^2 + 2L_1L_2\sigma/\rho \pi b_1b_2 + L_2^2/\rho \pi b_2^2.$$  

The interference factor $\sigma$ was computed by integrating the downwash associated with the larger wing over the span of the smaller wing’s wake in the Trefftz plane and so $\sigma$ was a function only of the span ratio and the vertical gap between the wings. If the two wings had the same span, carried the same lift, and were separated by a very large distance, the induced drag of the system approached 50% of the drag of a monoplane with the same total lift and span. As the wings were brought closer together, they would behave as a single wing with an overall span efficiency of 1.0. It is interesting that with zero vertical gap or with an infinite vertical gap, the optimal loading on each wing of a biplane with $b_1 = b_2$ is elliptical. In general,
we require equal and constant downwash on each wing of the arrangement so the optimal loading for nonzero, but finite, gaps would not be elliptical. This was of little significance for most biplanes, and tables of $\sigma$ values were used by aircraft designers to estimate the biplane-induced drag. The savings could be substantial, as suggested by Figure 6, although the high parasite drag of struts and the cable bracing dominated the drag picture. Now that cantilever structures can be built efficiently, one might ask if a modern, clean biplane arrangement might provide some drag reduction opportunities. This was suggested by Lange et al (1974), and designs such as Rutan’s Quickie (Downie 1984) have achieved some success. However, practical considerations such as fuel volume, structural weight, and lower Reynolds number generally overwhelm potential vortex drag advantages.

A more common example of a modern biplane is the conventional wing plus horizontal tail geometry. Such configurations can be analyzed with Prandtl’s biplane equation, and this has become a popular approach to the estimation of inviscid trim drag for wing-tail combinations. In the case of a coplanar wing and tail, the vertical gap is 0 and the $\sigma$ in Prandtl’s equation approaches $\sigma = \frac{b_2}{b_1}$, where $b_1$ is the span of the larger surface. This approach was initially applied to canard configurations as well, often showing large induced drag penalties because stability and trim considerations produced more than optimal lift on the smaller span’s forward surface. Researchers were surprised to find that in experimental tests, canard designs performed much better than predicted (Butler 1982). This was due to the fact that the actual load distribution on the wing of a canard configuration was not elliptical but rather was closer to the optimal load distribution in this case. Modifications to the biplane equation, based on optimal rather than elliptical load, provided much better comparison with experimental data and yielded the correct result—that for two coplanar surfaces, the minimum total induced drag depends only on the maximum span and total lift. That the inviscid trim drag for conventional or canard designs could approach zero in theory by the correct choice of wing lift distribution was obvious in view of Munk’s stagger theorem. It was also noted that even with elliptical loading, the same cancellation of trim drag could be achieved with a three-surface configuration—a downloaded aft tail canceled the wake of the lifting canard. This result led to some configuration studies, but the result in each of these cases was not very robust—small vertical gaps and the initial wake roll-up prevented complete cancellation of wakes, and drag rose very quickly with gap in real situations. Moreover, in the canard case, the optimal wing load distribution produced larger root-bending moments than the elliptical distribution, and when structural weight was included in the optimization, aft tail designs showed advantages for stable two-surface combinations in terms of total drag (McGeer & Kroo 1983).

Another biplane-related concept, in which the second wing of the biplane is a virtual image of the main wing, involves the exploitation of ground effect. Again the Prandtl biplane equation or a variant can be used to evaluate the total induced drag with one-half of the total associated with the actual wing. For most aircraft, the importance of ground effect is not great over much of the flight, but aircraft designed
CONCEPTS FOR INDUCED DRAG REDUCTION

605
to exploit the effect are intriguing. Large transport aircraft utilizing ground effects have been studied by Lockheed Georgia (Lange & Moore 1980), and applications to trans-Pacific cargo aircraft have been studied recently. Interesting work on such wing-in-ground effect aircraft in Russia is now becoming better known through technical publications (see Besyadovskiy et al 1993) and conferences devoted to the idea. This concept is exploited by birds, including pelicans, which achieve large drag reductions by flying at very small distances above the water surface (Hainsworth 1988).

Certain biplane concepts exploit some more subtle aspects of the aerodynamics of biplanes to achieve induced drag reduction. The keel of the recent Swiss entry in the America’s Cup yacht race incorporated two wings in tandem (Figure 7). The leeway angle of the hull and the motion of the wake of the forward surface leads to the development of an effective vertical gap between the Trefftz plane wakes of the two surfaces and a biplanelike drag reduction. It was noted that these boats were quite fast but suffered from stability and control problems.

Formation flight represents yet another instance of induced drag reduction with multiple lifting surfaces, although in this case the reduction is relative to the surfaces in isolation and is maximized in a planar configuration. The interaction may be analyzed in just the manner used to derive the Prandtl biplane equation, integrating the effect of one elliptically loaded wing over the others. The drag of the system may then be computed as a function of tip spacing and number of wings. Lissaman & Shollenberger (1970) showed that such results applied to the formation flight of birds by illustrating how the induced drag of a group of 25 identical birds might be reduced by as much as 60% over the total induced drag of the isolated birds flying at the same speed. This drag reduction is independent of the longitudinal distribution of the flock (according to the linear theory leading to Munk’s stagger theorem), and the V-like formation simply distributes the savings equally among the members of the flock. Drag reduction due to formation flying has been known for many years but is receiving renewed interest with the advent of automated precision navigation. With centimeter-level active positioning, the concept of multiple autonomous air vehicles, which takeoff independently and then assemble into formation for efficient cruising, becomes conceivable. If the wings need not be identical, the drag savings can be further enhanced by distributing the devices so that the overall lift distribution is approximately elliptical (Blake & Multhopp 1998). Physical connection of multiple vehicles has been investigated for a number of years and is now being discussed for application to micro-air vehicles and long-endurance autonomous aircraft to improve range.

Lastly, multiple, but nonparallel, lifting surfaces may provide some induced drag reduction and favorable interference. Lifting struts have been studied as part of a nonplanar lifting system and provide small potential induced drag benefits, although these benefits are probably insignificant compared with the drag reduction associated with increased span. Favorable interference between horizontal and vertical tail components is useful mostly to increase the lift curve slope, although engine-out, vertical tail induced drag can be important and is strongly affected by
Figure 7  Keel arrangement for recent racing sailboat (Swiss America's Cup entry 1999).
horizontal tail location (Katzoff & Mutterperl 1941). Even small devices such as flap hinge covers have been observed to produce a favorable and measurable change in aircraft drag through their interaction with the wing vortex wake (M Page, personal communication).

Wing Tip Devices

Because of the concentration of vorticity near the wing tip, devices to redistribute and interact with the vorticity in this region have been studied since the introduction of finite wing theory. Although low aspect ratio end plates were originally thought to retard the formation of tip vortices, the operation of such devices is now more commonly understood through the interaction with wake vorticity. As an example of the power of this far-field approach, we consider vertical winglets. The vortex drag reduction can be explained by a reduction in average wing downwash as the winglet moves much of the shed vorticity away from the wing plane. The sidewash created by the wing’s wake also modifies the local flow on the winglet and can lead to winglet thrust, whereas the winglet wake induces velocities on the wing. For an optimally loaded winglet, however, the far-field normalwash must vary with the cosine of the local dihedral, leading to constant downwash over the wing and zero net sidewash in the winglet region. This means that an optimally loaded system produces no net thrust or vortex drag on the winglet itself but rather a reduced wing downwash and drag. (This is strictly true only for unswept, optimally loaded systems, but the approach leads to the correct load distribution for general systems.) The drag reduction is directly related to the shape and extent of the vortex wake. Small tip devices are not able to eliminate or diffuse the wing vortex wake, and, just as for span extensions, the reduction in induced drag is strongly related to the additional bending moments added to the wing.

Figure 5 shows that a vertical surface located at the wing tip is worth approximately 45% of its height as additional span, if optimally loaded. To produce a large change in the vortex drag without a large increase in wetted area, low-aspect-ratio end plates were replaced with higher-aspect-ratio surfaces, termed winglets by Whitcomb, who provided some of the early experimental data and practical design guidelines for such devices (Whitcomb 1976). When the geometric span of the wing is constrained, well-designed winglets do provide significant reductions in airplane drag, and they have now been incorporated on aircraft ranging from sailplanes to business jets and large commercial transport planes. The justification for winglets, as opposed to span extensions, for aircraft that are not explicitly span-limited is less clear. Studies at NASA Langley that compared these two concepts with a constrained root-bending moment concluded that winglets were preferred over span extensions (Heyson et al 1977). Studies with somewhat different constraints suggested that the two approaches were almost identical in these respects (Jones & Lasinski 1980). The currently accepted view is that the complexity of the structural model and constraints limits the general applicability of any such conclusions. The evaluation of optimal winglet height and dihedral...
depends on the details of the wing structure, whether the wing is gust critical or
maneuver critical, whether large regions of the wing are sized based on a minimum
skin gauge, and whether the design is new or a modification of an existing design.
The evaluation of wing tip device advantages must be undertaken for each design
and include an array of multidisciplinary considerations. These include the effect
on aeroelastic deflections and loads, flutter speed, aircraft trim, stability and con-
trol effects (especially lateral characteristics), and off-design operation, as well as
effects on maximum lift, and finally, marketing considerations.
All of these effects have led designers to adopt large winglet surfaces, such
as on some canard designs, or very small winglets that produce small changes in
aircraft properties but provide a small benefit, or to not use these devices at all.
There is no clear answer to the optimal configuration, and even when winglets
are adopted, the geometries vary widely (Figure 8). The MD-11 uses a winglet
not unlike that described by Whitcomb (1976). The large upper element provides
vortex drag reduction without causing ground clearance problems. When the large
upper winglet is mounted aft on the upper surface, some of the leading edge
interaction is reduced—important under high $C_L$ conditions—whereas the small
lower winglet can be canted outward, reducing some of the effective dihedral
added by the upper element and reducing the torsional inertia, which is important

Figure 8  Winglet and wing tip device geometries.
in minimizing the impact on flutter speed. The wing tip fences incorporated on the A310 and other Airbus aircraft are very small by comparison, with less impact on the overall aircraft design. These are said to reduce aircraft drag by 1.5% in cruise (Poisson-Quinton 1985).

An interesting variant on the winglet concept involves staggering the vertical surface longitudinally. This “vortex diffuser” concept was studied by Lockheed in the 1980s (Hackett 1980), and although it is recognized that the vortex drag reduction is independent of the longitudinal position, some advantages are claimed for this aft positioning of the nonplanar surface. As the vertical surface is moved aft, the effect of the wing on the additional surface increases while the interference on the wing is reduced. This alleviates some of the difficulties with transonic and viscous interactions between winglet and wing, but it also points out limitations of Trefftz plane analysis and the stagger theorem. The minimum induced drag is independent of the longitudinal position of the lifting elements only if the optimal circulation distribution can be achieved. If elements are moved longitudinally, the twist of the surfaces must be changed to maintain the ideal circulation distribution. This may lead to difficulties with off-design operation if the twist is too large, or it may in fact be unachievable with any amount of twist.

Wing tip sails (Spillman 1978), consisting of multiple, high-aspect-ratio lifting elements attached to the wing tip at several dihedral angles, are another oft-cited variant of the winglet. Often analyzed in the near field, they may also be studied by more conventional far-field methods, although, as discussed subsequently, they may be more sensitive to wake geometry assumptions. When analyzed by the same Trefftz plane optimization approach as that used to construct Figure 6, these multiple surfaces appear less effective than a single vertical winglet with the same total span and vertical extent but may benefit from reduced transonic and viscous interactions at the intersection.

As part of an investigation into higher-order effects on vortex drag, Smith (1996) considered an entirely planar wing with multiple elements at the tip (Figure 9). The idea here was to introduce a wake geometry that looked very much like that associated with tip sails or two winglets with moderate dihedral. The sweep of the wing’s trailing edges introduced wake dihedral at angle of attack, even when the wake was assumed to be streamwise. Figure 9 shows the planform shape and the shape of the wake trace when the wing is at a 9° incidence and the wake is assumed to be streamwise. Based on this wake shape, an induced drag savings of about 5% is possible under these conditions. Of course the wake does not trail from the wing in the streamwise direction, and careful computation of the rolled-up wake geometry and inviscid drag shows that the effect of wake roll-up is to roughly double the gain expected for the streamwise wake. This 11% increase in span efficiency was significant, and the concept was studied in more detail both experimentally and computationally. Figure 10 shows the computed wake geometry and wing paneling used to compute vortex drag with the high-order panel code A502. The concept was fabricated and tested at NASA’s Ames Research Center. Data from the computations, balance, and detailed wake surveys
Streamwise Wake Shape at $\alpha = 9$ deg

Figure 9  Split tip planform and linear wake projection.

Figure 10  Split tip panel and computed wake geometry.
were found to agree well, showing 10–11% reductions in vortex drag due to these higher-order effects. The results are intriguing, and although the configuration was selected to exaggerate a particular effect rather than to serve as a good airplane wing, its potential application to aircraft is interesting.

With the myriad possible geometries for nonplanar wing design, one might attempt to find the “best” shape in a systematic manner. Although the computation of optimal loading is easily accomplished for a given wake geometry by using linear theory, the optimization of the wing topology (biplane, spanwise camber, box plane, winglet, and tip sails) is a very difficult problem owing to the complexity of the design space and the presence of multiple local minima. Application of a variable complexity evolutionary algorithm (Gage 1994) produced some interesting results, as shown in Figure 11. The system was allowed to build wings of many individual elements with arbitrary dihedral and optimal twist. Figure 11 shows

![Evolution of C-wing geometry](image_url)

**Figure 11** Evolution of C-wing geometry.
front views of the population of candidate designs as the system evolves, along with the best individual from a given generation. The system discovers winglets and then adds a horizontal extension to the winglet, forming a C-like shape. As shown in Figure 6, this concept achieves very nearly the maximum induced drag reduction associated with a box plane, but eliminates much of the area that would be required to close the box. Further studies showed that the optimal loading on the horizontal extension was downward, reducing the root-bending moment and providing a positive pitching moment when incorporated on an aft-swept wing. This led to some interesting studies by Boeing of the application of the concept to very large aircraft (McMasters & Kroo 1998).

In addition to these fixed tip devices, several studies of the use of rotating systems, such as propellers (Rubbert 1992) or turbines (Patterson 1985) at the tips of wings, have been performed. Rotating surfaces, like fixed surfaces, can be used to redistribute the vorticity in the trailing wake, and although the results are a bit more difficult to analyze, the same far-field approach can be applied. Such devices can be used to extract power, provide thrust, or reduce wing drag. Brief studies by the author have failed to show an advantage of such devices over winglets or span extensions with similar dimensions or bending moment increments, although one might imagine practical advantages for a single-bladed rotor that is stored conveniently behind the trailing edge on the ground.

Closed Systems

One can, of course, eliminate the wing tips altogether with configurations such as the ring wing (Terry 1964), box plane (Miranda 1972), joined wing (Wolkovitch 1986), or spiroid tips (Gratzer & Clark 1999) (For additional information, see http://www.aviationpartners.com/What_s_New/Spiroids/spiroids.html). Although the concept of eliminating the influence of tip vortices with these devices is ill conceived, such configurations do possess some interesting properties. The box plane achieves the minimum induced drag for given lift, span, and vertical extent, and a ring wing or joined wing also can achieve span efficiencies of >1.0 due to its nonplanar geometry, but no particular advantage is seen because these configurations are closed. The one feature apparent in this case, though, is that the optimal load distribution is not unique. One may superimpose a vortex loop with constant circulation on any of these wing geometries. This changes the local loading, but because the circulation is constant, the wake (and hence the lift and drag) is unchanged. While this does not reduce the vortex drag for a specified lift, it does provide some design flexibility. The optimal lift distribution of the box plane is generally shown as two horizontal wings that carry the same lift, connected by vertical planes whose circulation goes to zero at their midpoint (von Karman & Burgers 1935). We can, however, add a fixed circulation to the system so that the lower wing carries the entire lift and the upper wing carries none. The lift and vortex drag are unchanged. This is the reason that the C-wing geometry
CONCEPTS FOR INDUCED DRAG REDUCTION

shown in Figure 6 so closely approximates the drag of the box plane; we simply adjust the constant circulation increment so that the inner part of the upper wing is not needed. When applied to the joined wing configuration, whose upper and lower surfaces are staggered longitudinally, the constant circulation loop can add an arbitrary pitching moment to the design, providing trim over a wide range of lift centroid positions without a vortex drag penalty. When adapted to a winglet or system of wing tip devices—as on the spiroid tip or WingGrid (LaRoche & Palffy 1998)—this idea would not seem to provide induced drag advantages but might be used to moderate the local lift coefficients in favorable ways.

Other Related Issues

The topic of induced drag reduction is very broad. This review has highlighted a few interesting ideas, but there are many related areas that are topics in themselves. In the hope of stimulating the interested reader to pursue these areas, a few of these are mentioned briefly below.

The induced drag of unsteady systems is of interest in studies of aircraft in a turbulent atmosphere and aircraft with time-dependent control deflections as well as for flapping devices. Hall & Hall (1996) provide a nice analysis of flapping flight that includes far-field vortex drag estimation along with the effects of large amplitude motion (which leads to a nonplanar wake) and large changes in local dynamic pressure.

The vortex drag of low-aspect-ratio wings is complicated by leading and or side edge separation, which may lead to the loss of leading edge suction but also produces a highly nonplanar wake. The analysis of such situations is complex, but opportunities for drag reduction are often greater than those for the high-aspect-ratio wings, on which the present discussion focuses (Bushnell 1992).

The role of induced drag in maneuvering flight, both unsteady and quasi-steady, is a topic of interest with regard to airplanes ranging from pylon racers to fighters. Even the design of racing aircraft, which fly at speeds for which the induced drag in cruise is all but negligible, is strongly affected by induced drag considerations in turns.

Wing/body interference can have an important effect on vortex drag, not only through the change in wingspan loading but also because the far-field wake shape is affected by the presence of the fuselage. This latter effect leads to 2–3% penalties in the vortex drag of typical transport aircraft (Shevell 1983) and is sometimes taken into account in the design of winged keels for sailboats (van Oossanen & Joubert 1986; see also Figure 8).

Additional drag reduction concepts, including systems that involve integration of propulsion systems with the lifting surfaces, may lead to important drag savings in the future. These are often difficult to analyze, but the finite inviscid power required for a rotorcraft in hover demonstrates that the problem is not completely specified by lift, span, and forward speed.
CONCLUSIONS

Almost a century of research on the prediction and reduction of wing vortex drag has resulted in very accurate methods for drag estimation and a rather thorough understanding of the relevant fluid mechanics. A wide range of ideas for induced drag reduction have been suggested, many of which provide opportunities for significant drag reduction. The remaining challenge is to properly evaluate the implications of these concepts in the context of the complete aircraft system.

Although no single drag reduction device appears to offer clear and large advantages relative to simple wings, the potential for significant improvements through small steps now exists. Larger improvements may be possible indirectly, through active load management, constructive use of aeroelastics, and application of improved multidisciplinary optimization techniques early in the design process.

Visit the Annual Reviews home page at www.AnnualReviews.org

LITERATURE CITED


Eppler R. 1987. Die Entwicklung der
Tragflugeltheorie. Z. Fluwiss. Weltraumforsch. 11:133–44


CONTENTS

James Lighthill and His Contributions to Fluid Mechanics, *TJ Pedley* 1
Steady Streaming, *N Riley* 43
On the Fluid Mechanics of Fires, *Sheldon R Tieszen* 67
Experiments on Thermocapillary Instabilities, *Michael F Schatz and G Paul Neitzel* 93
Robert Legendre and Henri Werlé: Toward the Elucidation of Three-Dimensional Separation, *Jean M Delery* 129
Surface Pressure Measurements Using Luminescent Coatings, *James H Bell, Edward T Schairer, Lawrence A Hand, and Rabindra D Mehta* 155
Rossby Wave Hydraulics, *ER Johnson and SR Clarke* 207
Spin-Up of Homogeneous and Stratified Fluids, *PW Duck and MR Foster* 231
Extrusion Instabilities and Wall Slip, *Morton M Denn* 265
Turbulent Relative Dispersion, *Brian Sawford* 289
Early Work on Fluid Mechanics in the IC Engine, *John L Lumley* 319
Mechanics of Coastal Forms, *Paolo Blondeaux* 339
Aerodynamics of High-Speed Trains, *Joseph A Schetz* 371
Junction Flows, *Roger L Simpson* 415
Compression System Stability and Active Control, *JD Paduano, EM Greitzer, and AH Epstein* 491
Spilling Breakers, *JH Duncan* 519
Drag Due to Lift: Concepts for Prediction and Reduction, *Ilan Kroo* 587
Inertial Effects in Suspension and Porous-Media Flows, *Donald L Koch and Reghan J Hill* 619
### INDEXES

<table>
<thead>
<tr>
<th>Index</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subject Index</td>
<td>649</td>
</tr>
<tr>
<td>Cumulative Index of Contributing Authors, Vols 1–33</td>
<td>675</td>
</tr>
<tr>
<td>Cumulative Index of Chapter Titles, Vols 1–33</td>
<td>682</td>
</tr>
</tbody>
</table>