Improved Prediction of Turbomachinery Flows Using Near-Wall Reynolds-Stress Model

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In this paper an assessment of the improvement in the prediction of complex turbomachinery flows using a new near-wall Reynolds-stress model is attempted. The turbulence closure used is a near-wall low-turbulence- Reynolds-number Reynolds-stress model, that is independent of the distance-from-the-wall and of the normal-to-the-wall direction. The model takes into account the Coriolis redistribution effect on the Reynolds-stresses. The five mean flow equations and the seven turbulence model equations are solved using an implicit coupled \(O(\Delta x^3)\) upwind-biased solver. Results are compared with experimental data for three turbomachinery configurations: the NTUA high subsonic annular cascade, the NASA 37 rotor, and the RWTH 1 1/2 stage turbine. A detailed analysis of the flowfield is given. It is seen that the new model that takes into account the Reynolds-stress anisotropy substantially improves the agreement with experimental data, particularly for flows with large separation, while being only 30 percent more expensive than the \(k-\varepsilon\) model (thanks to an efficient implicit implementation). It is believed that further work on advanced turbulence models will substantially enhance the predictive capability of complex turbulent flows in turbomachinery. [DOI: 10.1115/1.1426083]

Introduction

Computational Fluid Dynamics (CFD) coupled to turbomachinery specific steady [1–3] and unsteady [4–6] models, and supported by carefully planned experiments [7–9], has greatly enhanced our understanding of the complex flow phenomena encountered in multistage turbomachinery [10,11]. There are three major research areas where progress is necessary for improving the predictive capability of computational methodologies:

1. Correct modeling of steady and unsteady multistage effects [11,12]
2. Inclusion of technological details [13–15], which is mainly a multiblock structured or unstructured grid management issue
3. Turbulence and transition modeling [16–18]

An examination of computational methodologies for steady and unsteady turbomachinery flows (Table 1) indicates that the Boussinesq hypothesis of tensorial proportionality between the Reynolds-stresses and the mean flow rate-of-deformation tensor [16] is almost invariably used, the more advanced models solving two transport equations (an equation for the turbulence kinetic energy and an appropriate scale-determining equation). Although two-equation models give better results than mixing-length models (and are independent of grid topology), they do not take into account the anisotropy of the Reynolds-stress tensor. More importantly they ignore the misalignment of the Reynolds-stress tensor and the mean flow rate-of-deformation tensor, which can be important in complex three-dimensional separated flows. Numerous variants of two-equation models exist, but globally results are very similar between variants. In order to improve upon the two-equation family, it seems necessary to use models that handle properly the Reynolds-stress tensor anisotropy. To the authors knowledge such models have not yet been evaluated for three-dimensional turbomachinery applications.

The Reynolds-stress models (RSM) are seven-equation closures, solving six transport equations for the six components of the symmetric Reynolds-stress tensor, and one scale-determining equation [65–67]. An additional interest of these models for turbomachinery applications is that the transport equations for the Reynolds-stresses contain exact Coriolis redistribution terms, and as a consequence take naturally into account the effect of rotation on turbulence. Recently, a Reynolds-stress closure for compressible separated flows, that is independent of the distance-from-the-wall and of the normal-to-the-wall direction, and that includes near-wall terms allowing integration to the wall, has been developed [68] and validated for a number of configurations [70,71]. This closure has also been extended to rotating flows [69].

The purpose of the present work is to examine the predictive capability of this RSM closure for turbomachinery configurations, and to assess potential improvements compared to two-equation closures. Results are presented for three turbomachinery configurations:

1. The NTUA subsonic (\(M<0.7\)) annular cascade [72–74], a stator with thin rotor-like profiles, subjected to inflow with important radial gradients and exhibiting a large separation at the hub, that computations using the Launder-Sharma \(k-\varepsilon\) closure fail to predict.
2. The NASA 37 rotor a well-known turbomachinery test-case [75–78], for which mixing-length and two-equation closures fail to correctly predict the nominal-speed operating line.
3. The RWTH 1 1/2 stage turbine [79,80], for which results using the Launder-Sharma \(k-\varepsilon\) closure show very good agreement with measurements.

Turbulence Model

The mean flow equations and the turbulence closure used in the present work are described in detail by Gerolymos and Vallet [68,69], and are summarized in the following for completeness. The transport equations for the mean-flow (Eqs. (1)(3)), and the Reynolds-stresses (Eq. 4), are written in a Cartesian reference-frame rotating with constant (time-independent) rotational velocity \(\Omega = \Omega e_r\).
Table 1  Turbulence models used in three-dimensional turbomachinery CFD

<table>
<thead>
<tr>
<th>Authors</th>
<th>Date</th>
<th>Closure</th>
<th>Model</th>
<th>Space</th>
<th>Time</th>
</tr>
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<tbody>
<tr>
<td>Adamczyk et al.</td>
<td>1990</td>
<td>2-eq</td>
<td>$k-e$ [27]</td>
<td>$O(\Delta x^2)$ upwind</td>
<td>implicit PB</td>
</tr>
<tr>
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<td>1990</td>
<td>0-eq</td>
<td>ML [23]</td>
<td>$O(\Delta x^2)$ centered</td>
<td>RK+IRS</td>
</tr>
<tr>
<td>Lakshminarayana</td>
<td>1992</td>
<td>2-eq</td>
<td>$k-e$ [27]</td>
<td>$O(\Delta x^2)$ centered</td>
<td>RK+IRS</td>
</tr>
<tr>
<td>Ameri et al.</td>
<td>1993</td>
<td>2-eq</td>
<td>$k-e$ [38]</td>
<td>$O(\Delta x^2)$ centered</td>
<td>RK+IRS</td>
</tr>
<tr>
<td>Hirsch et al.</td>
<td>1993</td>
<td>0-eq</td>
<td>ML [23]</td>
<td>$O(\Delta x^2)$ centered</td>
<td>RK+IRS</td>
</tr>
<tr>
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<td>1993</td>
<td>0-eq</td>
<td>ML [23]</td>
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<td></td>
</tr>
<tr>
<td>Turner and Jenning</td>
<td>1993</td>
<td>2-eq</td>
<td>$k-e$ WF [46]</td>
<td>$O(\Delta x^2)$ centered</td>
<td>RK</td>
</tr>
<tr>
<td>Vogel et al. [47,48]</td>
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<td>RK</td>
</tr>
<tr>
<td>Ameri et al. [50,51]</td>
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<td>2-eq</td>
<td>$k-e$ [49]</td>
<td>$O(\Delta x^2)$ centered</td>
<td>RK+IRS+MLTGRD</td>
</tr>
<tr>
<td>Furukawa et al. [52]</td>
<td>1998</td>
<td>0-eq</td>
<td>ML [23]</td>
<td>$O(\Delta x^2)$ centered</td>
<td>RK+IRS+MLTGRD</td>
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<td>$k-e$ WF [46]</td>
<td>$O(\Delta x^2)$ centered</td>
<td>implicit PB</td>
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<tr>
<td>Gerolymos and Vallet [55–57]</td>
<td>1998</td>
<td>2-eq</td>
<td>$k-e$ [58]</td>
<td>$O(\Delta x^2)$ centered</td>
<td></td>
</tr>
<tr>
<td>Arima et al. [59]</td>
<td>1999</td>
<td>2-eq</td>
<td>$k-e$ [27]</td>
<td>$O(\Delta x^2)$ centered</td>
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<tr>
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<td>$O(\Delta x^2)$ centered</td>
<td>RK+IRS</td>
</tr>
<tr>
<td>Sayma et al. [63]</td>
<td>2000</td>
<td>1-eq</td>
<td>1-eq [64]</td>
<td>$O(\Delta x^2)$ centered</td>
<td>implicit PB</td>
</tr>
<tr>
<td>present</td>
<td>2000</td>
<td>7-eq</td>
<td>RSM [68,69]</td>
<td>$O(\Delta x^2)$ centered</td>
<td></td>
</tr>
</tbody>
</table>

WF = wall functions; IRS = implicit residual smoothing; PB = pressure-based; RK = Runge-Kutta; MLTGRD = multigrid; ML = mixing-length; ARSM = algebraic Reynolds-stress model; RSM = Reynolds-stress model; † unstructured.

\[
\frac{\partial \bar{p}}{\partial t} + \frac{\partial \bar{p} \bar{W}_i}{\partial x_i} = 0
\]

\[
\frac{\partial \bar{p} \bar{W}_i}{\partial t} + \frac{1}{\partial x_j} \left[ \bar{p} \bar{W}_j \bar{W}_i + \bar{p} \delta_{ij} - \frac{1}{\partial x_j} \left( \bar{p} \bar{h}_i + \frac{1}{2} \Omega^2 \bar{R}^2 \right) \right] = 0
\]

\[
\frac{\partial}{\partial t} \bar{h}_i - \frac{1}{\partial x_j} \left( \bar{p} \bar{h}_j + \frac{1}{2} \Omega^2 \bar{R}^2 \right) = 0
\]

\[
\frac{\partial}{\partial \bar{W}_i} \left( \bar{W}_i \bar{W}_j \right) - \bar{p} \delta_{ij} + \frac{\partial}{\partial x_j} \left( \bar{p} \bar{W}_i \bar{W}_j \right) - \frac{\partial}{\partial x_j} \left( \bar{p} \bar{W}_i \bar{W}_j \right) + \frac{\partial}{\partial x_j} \left( \bar{p} \bar{W}_i \bar{W}_j \right) - \frac{\partial}{\partial x_j} \left( \bar{p} \bar{W}_i \bar{W}_j \right)
\]

where $t$ is the time, $x_i$ the cartesian space coordinates in the relative frame-of-reference, $\nu_{ijk}$ the third-order antisymmetric tensor [81], $\delta_{ij}$ the Kronecker symbol [81], $R$ the radius (distance from the axis of rotation: $R^2=[x_i-x_i \Omega^2 \Omega_i \Omega_j]x_j - [x_i \Omega^2 \Omega_j x_j]$), $W_i$ the relative velocity components, $V_i=W_i+e_{ijk} \Omega_j x_k$ the absolute velocity components, $\rho$ the density, $p$ the pressure, $\tau_{ij}$ the viscous stresses, ($\bar{\cdot}$) Favre-averaging, ($\cdot$) nonweighted-averaging, ($\cdot^*$) Favre-fluctuations, ($\cdot^*$) nonweighted-fluctuations, $\bar{h}_i=\bar{h}+1/2 \bar{W}_i \bar{W}_i$ the total enthalpy of the relative mean flow (which is different from the Favre-averaged total en-
The model coefficients \( C_1, C_2, C_{1n}, C_{2n} \) are functions of the anisotropy tensor \( A_{ij}, A_{ij} \) and of the turbulence-Reynolds-number \( Re_T \) (Table 2). The pseudonormal direction \( n \) appearing in the echo terms is given by the direction of the gradient of a function of turbulence length-scale \( l_T \) and of the anisotropy tensor variables (Table 2).

The dissipation-rate of the turbulence-kinetic-energy \( e \) is estimated by solving a transport equation for the modified-dissipation-rate \( \tilde{e} = e - 2 \tilde{v} \cdot (\nabla \tilde{k})^2 \) (\( \tilde{v} \) the kinematic viscosity). The wall boundary-condition is \( \tilde{e}_w = 0 \), offering enhanced numerical stability.

The modelled Launder-Sharma [58] equation, with a tensorial diffusion coefficient [90] is used

\[
\frac{\partial \tilde{e}}{\partial t} + \frac{\partial}{\partial x_l} \left( \tilde{v} \tilde{e} \right) - \frac{\partial}{\partial x_k} \left[ \left( \tilde{u}_k \tilde{u}_k \tilde{e} \right) - \frac{k}{\rho} \frac{\partial \tilde{p}}{\partial x_l} \right] = C_{e1} \tilde{e} \left( \frac{\gamma}{\gamma - 1} \right) R^\frac{1}{\gamma - 1} \frac{2 \tilde{v} \cdot \nabla \tilde{k}^2}{\rho} \frac{1}{\tilde{v} \cdot \nabla \tilde{k}}
\]

The turbulent heat-flux \( \overline{\rho h w_i w_j} \) is closed by a simple gradient model [68]

\[
\overline{\rho h w_i w_j} = c_p \frac{\gamma}{\gamma - 1} R^\frac{1}{\gamma - 1} \frac{2 \tilde{v} \cdot \nabla \tilde{k}^2}{\rho} \frac{1}{\tilde{v} \cdot \nabla \tilde{k}}
\]

where \( c_p \) is the heat capacity at constant pressure, \( Pr_T \) the turbulent Prandtl number (in the present work \( Pr_T = 0.9 \) to obtain the correct recovery temperature for turbulent flow over an adiabatic wall), and \( Re_T \) the turbulence Reynolds number based on the modified dissipation \( \tilde{e} = e - 2 \tilde{v} \cdot (\nabla \tilde{k})^2 \) (\( \tilde{e} \) being turbulence-kinetic energy dissipation, and \( \tilde{v} \) the kinematic viscosity).
The thermodynamics of the working gas and the mean viscous stresses and heat-flux are approximated by standard closure assumptions [68,90,91]. The flow has to go around the separation bubble, and this results unfortunately no detailed measurements of the high values of 3–4 percent. The value of the Mach number plots show the large separation bubble, and this results in high outflow swirl at the hub (corresponding to substantial unsteady region, and as a consequence gave very poor agreement with measured outflow angles. The incoming flow is quite complex, because the swirl necessary to obtain the desired inlet flow-angle was experimentally obtained by using a scroll (and not static vanes). As a consequence inflow profiles of total-pressure $p_{iM}$ and flow-angle $\alpha_M$ contain important radial variations (Fig. 1). The turbulence intensity at inflow was experimentally estimated at the high values of 3–4 percent. The value $\tau_{\omega} = 4$ percent was applied as inflow condition in the computations (Table 3).

Comparison of computed and measured pitchwise-averaged quantities at inflow and outflow planes (Fig. 1) shows substantial differences between the present RSM and the Launder-Sharma $k - \varepsilon$ [58] predictions. These computations were run using grid-D of $2.3 \times 10^6$ points (Table 4). This is a rather fine grid with $n_x < 3.4$ everywhere. At the inflow plane (situated 0.2 axial chords $x_i$ downstream of the computational inflow plane where the inflow profiles are applied) it is seen that both models accurately simulate the radial distributions of $\sigma_M$ and $p_{\alpha_M}$. They show, however, a difference in the turbulence profiles near the hub, due to a different development from computational inflow downstream, the RSM computations predicting a lower level of turbulence near the hub (unfortunately no detailed measurements of $k_w$ were available). At the outflow the RSM computations correctly predict the experimentally measured high swirl near the hub. This swirl is associated with a large hub-corner-stall, on the suction-side of the blades (Fig. 2). The Mach-number plots show the large separation predicted by the RSM computations on the suction-side (Fig. 2). The flow has to go around the separation bubble, and this results in high outflow swirl at the hub (corresponding to substantial unsteady region, and as a consequence gave very poor agreement with measured outflow angles. The incoming flow is quite complex, because the swirl necessary to obtain the desired inlet flow-angle was experimentally obtained by using a scroll (and not static vanes). As a consequence inflow profiles of total-pressure $p_{iM}$ and flow-angle $\alpha_M$ contain important radial variations (Fig. 1). The turbulence intensity at inflow was experimentally estimated at the high values of 3–4 percent. The value $\tau_{\omega} = 4$ percent was applied as inflow condition in the computations (Table 3).

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the two models are also seen in the plots of turbulence-kinetic-energy $\kappa$ (Fig. 2), where one can also observe the larger wakes predicted by the RSM computations. Comparison of computed and measured total-pressure $P_{tot}$ distributions at cascade exit (Fig. 1) indicates good agreement. The RSM computations slightly overestimate losses near the hub. This, together with the slightly higher than measured values of $\alpha_M$, suggest that the present model slightly overestimates the separated flow region, a problem attributed rather to delayed reattachment than to excessive separation. It is indeed believed that the predicted separation is not too thick, but that it does not end as abruptly as it should.

In order to assert grid independence of the results, computations were run using different grids (Table 4). Grid B of $\sim 10^6$ points has 69 radial surfaces, and satisfactory $n_{r0}$ ($<0.7$), both on the flowpath walls and on the blades. Grid refinement strategy maintained the size of the first grid-cell away from the walls, by using more points with a lower stretching near the walls (geometric stretching was invariably used [92]). Grid D of $\sim 2.3 \times 10^6$ points has 141 radial stations, and slightly more blade-to-blade points (Table 4). Grid E of $\sim 3 \times 10^6$ points has the same radial resolution as grid D, but a finer blade-to-blade grid (81 points from the blade surface to mid-passage, corresponding to 161 points from one blade to its neighbor), in order to examine the influence of blade-to-blade refinement (Table 4).

It should be noted that both the radial refinement (grid B to

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**Table 4** Computational grid summary

<table>
<thead>
<tr>
<th></th>
<th>UH</th>
<th>O</th>
<th>DH</th>
<th>TC</th>
<th>OZ</th>
<th>points</th>
<th>$n^+_{r0}$</th>
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<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>grid B</td>
<td>$17 \times 47 \times 69$</td>
<td>$201 \times 49 \times 69$</td>
<td>$51 \times 51 \times 69$</td>
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<td>$914 \times 181$</td>
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<td>$201 \times 53 \times 141$</td>
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<td>$2 \times 269 \times 113$</td>
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<tr>
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<td>$81 \times 61 \times 101$</td>
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<tr>
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<td>$201 \times 17 \times 41$</td>
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<td>$201 \times 49 \times 121$</td>
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<td>$201 \times 31 \times 41$</td>
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</tbody>
</table>

UH = upstream-H-grid (axial×tangential×radial); O = blades-O-grid (around the blade×away from blade×radial); DH = downstream-H-grid (axial×tangential×radial); TC = tip-clearance-O-grid (around the blade×away from blade×radial); OZ = O-zoom-grid (around the blade×away from blade×radial); † without O-grid points overlapped by the OZ-grid; $n^+_{r0}$ on the blades; $n^+_{TP}$ on the flowpath.

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**Fig. 1** Comparison of measured and computed (using the present RSM and the Launder-Sharma $k-\varepsilon$ [58]) pitchwise-averaged flow-angle $\alpha_M$, total-pressure $P_{tot}$, and turbulence-kinetic-energy $k_M$ for the NTUA 1 annular cascade ($\dot{m} = -13.2 \text{ kg s}^{-1}$; $T_{in} = -4\%$; grid D).

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**Fig. 2** Comparison of measured and computed (using the present RSM and the Launder-Sharma $k-\varepsilon$ [58]) pitchwise-averaged flow-angle $\alpha_M$, total-pressure $P_{tot}$, and turbulence-kinetic-energy $k_M$ for the NTUA 1 annular cascade ($\dot{m} = -13.2 \text{ kg s}^{-1}$; $T_{in} = -4\%$; grid D).
grid_D) and the blade-to-blade refinement (grid_D to grid_E) are substantial (factor 2). Both the \( k-\varepsilon \) and the RSM computations (Fig. 3) indicate that doubling the number of points radially enhances the prediction of the separation region (2.5 deg in \( \alpha_M \) for the \( k-\varepsilon \), and 4.5 deg in \( \alpha_M \) for the RSM). The blade-to-blade refinement (grid_E) was investigated for both closures (Fig. 3), and results are identical with the results of grid_D. It is believed that grid_D is adequate, although computations with an even finer grid (radially) would be needed to demonstrate this assertion. It should be noted that even the coarse grid_B RSM computations are better than the fine grid_D \( k-\varepsilon \) (Fig. 3), underlining the substantial improvement in flow angle prediction by the RSM closure. This improvement is associated with a better prediction of the separated flow structure.

**Transonic Compressor Rotor.** The NASA 37 transonic rotor [75–78] is a well known turbomachinery test-case. Experimental data for the NASA 37 transonic rotor were obtained at various measurement planes, using both LDV (LASER Doppler Velocimetry) and classic rake measurements of \( p_t \) and \( T_t \). The measurements uncertainties were reported by Suder [78]: massflow \( \dot{m} \pm 0.3 \text{ kg s}^{-1} \); absolute flow angle \( \alpha \pm 1.0 \text{ deg} \); total pressure \( p_t \pm 100 \text{ Pa} \); total temperature \( T_t \pm 0.6 \text{ K} \).

Computations by numerous authors [13,26,32,56,59,77,102,103] using a wide variety of turbulence models and numerical methods, highlight the predictive CFD state-of-the-art for this configuration. A careful examination of the computations indicates that, in the limit of grid-converged results, both zero-equation and two-equation models overestimate the total-to-total pressure ratio \( \frac{p_t}{p_t} \) as a function of massflow \( \dot{m} \). The zero-equation models overestimate \( \frac{p_t}{p_t} \) by \( \approx 3 \) percent, whereas the two-equation models overestimate \( \frac{p_t}{p_t} \) by \( \approx 1.5 \) percent [103]. Grid convergence is important, as demonstrated by comparing the results using the \( k-\varepsilon \) model of Chien [27] obtained by Hah and Loellbach [26] using \( \approx 1.9 \times 10^6 \) points grid and by Arima et al. [59] using \( \approx 0.6 \times 10^6 \) points. The latter grid was particularly coarse in the blade-to-blade direction, and as a consequence underestimated choke massflow \( \dot{m}_{CH} \) (20.77 \text{ kg s}^{-1}).

**Fig. 2** Comparison of Mach-number \( \tilde{M} \) and turbulence-kinetic-energy \( k \) computed (using the present RSM and the Launder-Sharma \( k-\varepsilon \) [58]), at 25 percent span (\( \tilde{m}=13.2 \text{ kg s}^{-1} \); \( T_u=4 \text{ percent} \); grid_D).
instead of the measured value of 20.93 kg s\(^{-1}\), which was correctly predicted by the fine grid computations by Hah and Loellbach [26]. The associated increased blockage gave a seemingly good prediction of pressure-ratio in the coarse grid computations [59], but the characteristic is translated towards lower massflow (in terms of dimensional \(m\)), and the results are not representative of the grid-converged model performance.

If the form of the spanwise distribution of the pitchwise averaged total pressure \(p_{\text{M}}\) downstream of the rotor is considered, there are two regions of discrepancy with measurements: 1) a local peak of \(p_{\text{M}}\) near the casing, corresponding to a too strong tip-clearance vortex, and 2) a \(p_{\text{M}}\) deficit near the hub (this deficit is attributed to both an underestimation of hub-corner stall by the models [26] and to massflow leakage emanating from a small gap between the stationary and rotating parts of the hub upstream of the rotor [13] which was not modelled in the computations).

Previous studies by the authors [55,56] using the same grid-generation methodology [92] and the same numerical scheme, but with the Launder-Sharma \(k-\varepsilon\) turbulence model [58], include grid-convergence studies using 1, 2, and \(3 \times 10^6\) points (Table 3), indicating that results with grid C (2 \(\times 10^6\) points) are practically grid independent. Based on these results, all the computations presented here were run on grid D of \(3 \times 10^6\) points (Table 3). The computational grid consists of an H-O-H grid with 161 radial stations. Tip clearance is discretized using an independent O-type grid with 41 radial stations [55,92]. Comparison of the measured characteristic (\(\alpha_{\text{M}}\)) between stations 1 and 4 versus \(m\) at nominal speed (Fig. 4) with computations using the new RSM closure [69] and the Launder-Sharma \(k-\varepsilon\) turbulence model [58] indicate that the RSM results follow closely the experimental characteristic. The improvement of the agreement with measurements is substantial, compared to the \(k-\varepsilon\) results (Fig. 4). Examination of the spanwise distribution of pitchwise-averaged total pressure \(p_{\text{M}}\) at station 4, for various operating points shows that the improvement is mainly due to the accurate prediction between 40 percent and 80 percent span (Fig. 4), where the RSM results closely follow the experimental data, improving upon the \(k-\varepsilon\) computations. There is also noticeable improvement in predicting the \(p_{\text{M}}\) deficit near the hub (where the nonSimulated massflow leakage might account for the remaining discrepancy), for all operating points (Fig. 4). On the other hand, the RSM model fails to correct the parasite \(p_{\text{M}}\) peak near the casing, indicating that the relaxation behavior of the model must be improved. Comparison of computed and measured spanwise distributions of pitchwise-averaged absolute flow angle \(\alpha_{\text{M}}\) at station 4 for the different operating points (Fig. 4) shows good agreement between the two models and the experiment. The RSM results underestimate \(\alpha_{\text{M}}\) by \(1 \deg\), which is within measurement accuracy [78], whereas the \(k-\varepsilon\) results are very close to the experimental data.

In order to understand the mechanism responsible for the improved agreement with measurements, the isentropic Mach number distributions \(M_{\text{s}}\) [69] at 70 percent span (Fig. 5) are examined. At operating point 1, the RSM results predict a flow at the limit between started and unstarted regime [104], whereas the \(k-\varepsilon\) computations indicate that the flow is started, with a clearly visible pressure-side shock-wave (Fig. 5). On the suction-side the RSM results predict a shock-wave location \(5 \text{ percent } x\) further upstream compared to the \(k-\varepsilon\) computations (Fig. 5). This point is choked, so that the correspondence between experiment and computations is taken at the same pressure-ratio (and same mass-
flow), corresponding to different shock-structures in the two models. For all the other operating points the flow is unstarted [104], with the RSM results predicting the suction-side shockwave systematically ~5 percent $\chi_s$ ($\chi_s$=axial chord) upstream of the $k - e$ location. Similar conclusions are drawn at other spanwise locations. It is plausible that the main improvement brought by the RSM closure is an improved prediction of the limit between started and unstarted flow, attributed to a better prediction of blockage [78], because of a better prediction of shock-wave/ boundary-layer interaction. Another improvement of the RSM closure is a more pronounced $p_{tu}$ peak very near the hub (Fig. 4), for all operating points, indicating a better prediction of hub secondary flows.

**Turbine 1 1/2 Stage.** Finally computations were run for a 1 1/2 stage axial flow turbine, experimentally investigated at the Institut für Strahltriebe und Turboarbeitsmaschinen of the RWTH [79,80]. Steady three-dimensional multistage computa-
Fig. 5 Computed (using the present RSM and the Launder-Sharma $k-\varepsilon$ ([58])) isentropic-Mach-number distributions at 70 percent span for various operating points at design-speed, for NASA 37 rotor ($\dot{m} = 20.85, 20.79, 20.65, 20.51, 20.12, 19.78, 19.36$ kg s$^{-1}$; $T_u = 3$ percent, $\delta_{tc} = 0.356$ mm; grid D)
Fig. 6 Measured and computed (using the present RSM and the Launder-Sharma $k-\varepsilon$ [58]) radial distributions of pitchwise-averaged total pressure $p_t\bar{M}$ and flow angle $\alpha_M$ for RWTH_1 turbine 1 1/2 stage ($\dot{m}=8.23\text{ kg s}^{-1}$; $T_u=3\%$; $\delta_{TC}=0.4\text{ mm}$; grid_D).

Fig. 7 Computed entropy and turbulent kinetic energy plots at various axial planes in the rotor of the RWTH_1 turbine 1 1/2 stage ($\dot{m}=8.23\text{ kg s}^{-1}$; $T_u=3\%$; $\delta_{TC}=0.4\text{ mm}$; RSM grid_D).
tions for this configuration (Table 3) have been compared with measurements by Emunds et al. [105], who used a mixing-length turbulence model [23]. Volmar et al. [106] have performed unsteady computations with time-lagged pitchwise periodicity for this configuration, using a $k-e$ model [27]. Gallus et al. [107] have performed both steady and unsteady computations, for the stage without the outlet-guide-vane, using a $k-e$ model [27].

In the present work we have performed steady-multistage computations using four different grids of 1, 2, 3, and $4.4 \times 10^6$ points (Table 4) with 51, 65, 81, and 121 radial stations, respectively. The multistage method is based on a mixing-plane approach between blade-rows, and is described in detail in Gerolymos and Hanisch [57]. The meridional averages that are conserved across the interface are density, mass-weighed velocities, static pressure, Reynolds-stresses, and kinetic-energy-dissipation-rate [57]. The matching between rows is achieved using overlapping grids that allow matching of both values and through-flow-wise gradients of the conserved quantities, ensuring very good continuity at the interfaces [57].

Comparison of measured and computed pitchwise-averaged total pressure $p_{1M}$ and absolute flow-angle $z_M$ at various axial stations (Fig. 6) indicates that there is close agreement between the RSM and the $k-e$ computations on the fine grid_D ($4.4 \times 10^6$ points). Agreement with measurements is good for the flow-angles $z_M$, but the computations slightly overestimate the total pressure $p_{1M}$ at rotor exit (plane 2), and as a consequence at the stage exit plane 3. This overestimation corresponds to a $\sim 1.5$ percent underestimation of turbine expansion ratio.

The form of the radial distribution of $p_{1M}$ is nonetheless very well predicted (Fig. 6). Volmar et al. [106] note that there are some slight inconsistencies in the experimental data (measurements were taken at different planes for slightly different values of $\dot{m}$, and different values of inlet total pressure $p_{1w}$). In our computations the same problems were encountered. As was there some uncertainty concerning massflow, it was preferred to run the computations at a massflow $\dot{m}=8.23\, kg/s$ slightly higher than the average experimental massflow $\dot{m}_{exp}=8.8\, kg/s$, so as to have good agreement in rotor outflow (plane 2) angle $z_M$ (Fig. 6). This resulted in the slight difference in $p_{1M}$ level. This choice (instead of fitting $p_{1M}$ with a corresponding discrepancy in $z_M$) was taken because of our interest in the secondary flow phenomena at rotor exit (Fig. 7). Each peak on the rotor-exit $z_M$ (Fig. 6) distribution can be identified with a secondary flow-peak in entropy and turbulence-kinetic energy distributions (Fig. 7). The overall agreement with measurements is quite good, for both turbulence models, except at $\sim 20$ percent span, where a slight dip in $z_M$, associated with an important dip in $p_{1M}$, is not correctly predicted. Emunds et al. [105] argue that this location corresponds to the interaction between the nozzle-hub and the rotor-hub secondary vortices.

The grid influence on results is illustrated by comparing the results obtained using the different grids (Table 4) and the two turbulence models for the $z_M$ distribution at rotor exit (Fig. 8). Concerning the RSM computations, it is seen that the coarsest grid_A, with 51 radial stations fails to predict correctly the structure of the secondary flows. It should be noted that this grid has unacceptable high values of $n_s^+ = 5 - 10$ (Table 4). The RSM computations on grid_B with 65 radial stations and $n_s^+ = 3/2$ (Table 4) does a good job in predicting the structure of the secondary flows everywhere, except near the casing where it fails to correctly describe the tip-leakage vortex, associated with the $z_M$-peak at 96 percent span (Fig. 8). This is improved in the RSM computations on grid_C which has 81 radial stations, $n_s^+ < 1$, and a finer grid within the tip-clearance-gap (Table 4). This grid predicts the tip-leakage vortex $z_M$-peak at 96 percent span, but not the undulation at 90 percent span, corresponding to the interaction between tip-clearance and the casing secondary vortex (Fig. 8). Finally, the RSM computations on grid_D with 121 radial stations predict correctly the secondary flows. Examination of the $k-e$ computations on grid_B and grid_D reveals the interesting feature that although both models give similar results on the fine grid_D, the RSM computations are substantially better on the coarse grid_B, compared to the $k-e$ computations on the same grid (Fig. 8).

**Conclusions and Perspectives in Turbulence Modelling**

In the present work a new near-wall low-turbulence-Reynolds-number Reynolds-stress model (RSM), that has been designed to be completely independent of wall-topology (distance-from-the-wall and normal-to-the-wall orientation), has been evaluated by comparison with experimental measurements, and with results using the Launder-Sharma $k-e$ model, for three turbomachinery configurations. To the authors knowledge, this is the first time that a full near-wall second-moment closure is applied to complex three-dimensional turbomachinery configurations.

For the NTUA_1 subsonic annular cascade, the RSM closure corrects the deficiency of the $k-e$ model, by predicting the large suction-side hub-corner-stall observed experimentally. This results...
in a substantially improved prediction of cascade-exit flow-angle distribution, resulting from a better prediction of the complex three-dimensional separated flow structure.

For the NASA 37 transonic compressor rotor, the RSM closure improves the massflow versus pressure-ratio operating-map prediction, by improving the prediction of the radial distribution of total-pressure, through a better prediction of rotor-shockwave structure (and of shock-wave/boundary-layer interaction). In particular, the rotor spill-point (where the flow at the tip becomes unstalled) is correctly predicted.

For the RWT1 1.1/2 stage axial flow turbine, both models give good prediction of the flow, with the RSM model being less grid-sensitive than the k–ε model, an important advantage for industrial applications, on relatively coarse grids. In this case the $k$–$ε$ results on the fine grid are very satisfactory, because there is no substantial flow separation.

Globally, the present RSM closure yields invariably better results than the $k$–$ε$ closure, especially when flow separation dominates the flowfield. For flows with little separation, the improvement is marginal, but for all the configurations studied by the authors results are invariably better with the RSM closure. Experience with the model shows that it is as robust as the RANS closure, by improving the prediction of the radial distribution of turbulence structure. It is believed that further validation will considerably increase confidence in CFD results than 2-equation closures, and also provide a better description of turbulence structure.


